The Equilibrium Point Hypothesis for Control of Movements

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Introduction

If one was to take a robot arm and replace each of its motors with a pair of opposing rubber bands, the arm would tend to settle to the same configuration no matter from where it was released. That configuration is the equilibrium point of the system. If we now change the length-tension properties of the rubber bands, for example, change the resting lengths or stiffnesses, the equilibrium point of the system will change. Our muscles share a property with rubber bands in that the static force that they generate depends on length: the greater the length, the greater the force (see Muscle Models). The activations received by motor neurons, whether it be from direct descending commands from the brain or from the spinal reflex circuitry, can change the force-length relation for each muscle, resulting in a change in the equilibrium position of the system. When we reach for an object, is the smooth, stable motion a consequence of a simple trajectory of equilibrium points? Are our muscles and the associated spinal reflex circuitry designed in a way that makes control of motion particularly simple for the brain?

If the answer is yes, then it implies that much of the problems inherent in control of a multi-joint limb, for example, non-linear state-dependent dynamics, might be simplified because of a well designed muscle-reflex system. Here I review the evidence regarding this hypothesis.

Mathematical basis of the hypothesis

Equilibrium refers to a state of a system in which the forces acting on it are zero. For example, if the dynamics of the system are

$$\dot{q} = h(q, u) \tag{1}$$

where q is the state of the system and u(t) is a control input, then the equilibrium points q^* satisfy the following condition:

$$0 = h(q^*, u) \ \ \text{for all} \ \ t \geq t_0.$$

In short, if the system reaches an equilibrium position, it will remain there.

For a mechanical system, the state is an ordered pair $q = \{\theta, \dot{\theta}\}$, where θ and $\dot{\theta}$ are the position and velocity of the system. A change in the state occurs when there are forces acting on it. This can be written in the framework of Eq. (1) as

$$\ddot{\theta} = I(\theta)^{-1}(f_c(\dot{\theta}, \theta, u(t)) - f_m(\dot{\theta}, \theta)) \tag{2}$$

where I is the system's inertia, f_c is the external force field imposed on the system due to the controller with control input u(t), and f_m is the force field produced by the motion of the inertial coordinate frames (Coriolis and centripetal) and other forces. It follows that the system is at equilibrium at any state $\{\theta = 0, \dot{\theta} = 0\}$ where the force in the net field $f_c - f_m$ is zero. Any such position θ^* is an equilibrium point for the system.

We call each state where a field has zero force a *null point* of that field. The equilibrium points for the system, however, are a subset of these null points: the equilibrium point exists only at those null points of the force field $f_c - f_m$ where the state has zero velocity.

Let us consider how we could go about controlling the system of Eq. 2. Our objective may be to select the input u in such a way that the system follows a desired trajectory $\theta_d(t)$. For this to occur, we might select u at any time t in such a way that if we were at state $\{\theta_d, \dot{\theta}_d\}$, our controller would produce a force $f_c = \hat{f}_m + \hat{I}\ddot{\theta}_d(t)$, where $\hat{x}$ is the controller's estimate of x. Since there may be uncertainties in the environment, it's a good idea to also have a mechanism to push us toward where we should be if the need arises:

$$f_c = \hat{f}_m + \hat{I}\ddot{\theta}_d - B(\dot{\theta} - \dot{\theta}_d) - K(\theta - \theta_d) \tag{3}$$

where B and K should be positive definite matrices. We can think of the estimates as a feedforward component of the controller, and the remainder as the feedback component of the controller. If the estimates were perfect, substitution of Eq. 3 into Eq. 2 would give:

$$\ddot{e} + c_1\dot{e} + c_2 e = 0$$

where $e = \theta - \theta_d$ is the error in tracking our desired trajectory, and c_1 and c_2 are positive definite (because the inertia matrix I is also positive definite for a mechanical system). Therefore, the tracking error would exponentially decline with time and the system will be stable about the desired trajectory.

Eq. 3 makes plain the notion that the forces produced by the controller must take into account the system's mass if it is to move the system along the desired trajectory. The estimates are *internal models* that the brain would presumably have to know (see Motor Control, Biological and Theoretical for a discussion on how these models might be learned). However, the equilibrium point hypothesis suggests that the feedback system in Eq. 3 is designed in a way that largely eliminates the need for the estimates of the dynamics of the limb. In this hypothesis, the muscles and the spinal reflexes function as the feedback system about the desired trajectory, i.e., the stiffness and viscosity of the system. The main question is the extent to which the mechanical behavior of muscles and the reflex system can compensate for the dynamics of the limb.

Biomechanical behavior at rest

In a seminal paper by Feldman (1966), it was observed that the spinal control system acting on the elbow joint of the human arm (composed of muscles and the local feedback circuitry) had static characteristics that were similar to a non-linear spring. When the elbow was displaced from its equilibrium position, muscles produced monotonically increasing force (as measured at the hand):

$$f = a(\exp[b(x(t) - x_\lambda(t))] - 1) \tag{4}$$

where f is muscle force, t is time, $x(t)$ is length of a muscle, and $x_\lambda(t)$ is the threshold length beyond which the muscle will produce force. Feldman's thesis was that the signals sent from the brain to the spinal reflexes and muscles could be interpreted as setting the threshold length $x_\lambda(t)$ for each muscle. Feldman and Orlovsky (1972) later showed that stimulation of a motor center in the brainstem (of cats), resembling what might happen in a voluntary change in the brain's input to the spinal cord, did result in force-length changes in the muscles. These changes appeared as changes in $x_\lambda(t)$ in the above system.

For a constant input $x_\lambda(t)$ in Eq. 4, muscle force reflects both the mechanical properties of the isolated muscles (increased production of force when muscle is lengthened), and the effect of local neural feedback (recruitment of more motorneurons if length exceeds a set threshold). There is now independent support for the formulation in Eq. 4. Hoffer and Andreassen (1981) measured the rate of change in stiffness with respect to force in muscles of a cat's hindlimb. They found the relation between force and stiffness to be independent of muscle length, and of the form:

$$\frac{df}{dx} = k(1 - \exp[-\alpha f]) \tag{5}$$

where df/dx is muscle stiffness. Shadmehr and Arbib (1992) noted that the solution to the above differential equation has the form:

$$f = \frac{1}{\alpha} \ln(\exp[\alpha k(x - \lambda)] + 1) \tag{6}$$

In the above, λ is the constant of integration and depends on the initial conditions for Eq. 5. This result demonstrated that an intact muscle-reflex system has a static behavior that resembles a non-linear spring with an adjustable threshold.

If a single-joint limb is controlled by a pair of muscles, then setting λ for each muscle sets the equilibrium point of the system and describes a force field about this equilibrium. Hogan (1985) showed that in a multi-joint system, this field will be conservative. This means that if the nervous system produces a force field f_c in Eq. 3 through setting of threshold lengths for the muscles of the limb, then when $\ddot{\theta}$ and $\dot{\theta}$ are zero, curl of the field f_c should be zero. Mussa-Ivaldi, Hogan, and Bizzi (1985) measured the static component of f_c in humans. The procedure was to have subjects hold on to the handle of a robotic arm. The robot produced force perturbations at various directions and measured the steady-state force response of the subject's arm as a function of position. It was found that the resulting force field was essentially curl-free. Taken together, static behavior of muscles and the spinal control circuitry appeared to be well described as a non-linear spring with an adjustable threshold length.

Movements as a shift in equilibrium position

When threshold lengths are set for each muscle, the result is a corresponding equilibrium position θ^* for the limb. The major contribution of EPH has been to suggest that motion is generated by

the CNS through a gradual transition of equilibrium points along the desired trajectory without an explicit compensation for dynamics. The evidence for this initially came from a simulation study by Flash (1987). She suggested that in the case of human reaching movements in the horizontal plane, it was possible to predict the hand's motion accurately by smoothly shifting the equilibrium point along a straight line from the start point to a target location. Interestingly, she showed that in the simulation, because the controller was not attempting to compensate for the limb's dynamics, the hand's trajectory slightly deviated from a straight line. However, it turns out that the trajectories recorded in human subjects also show similar deviations, matching her simulations. In this model, the controller was composed of a linear spring-dashpot system with adjustable threshold:

$$f_c = K(\theta - \theta^*(t)) + B\dot{\theta}$$

The field had the property that its static behavior about equilibrium was defined by a stiffness matrix K. This matrix was measured about the equilibrium position of a resting arm by Mussa-Ivaldi, Hogan, and Bizzi (1985).

Taking a different approach, Shadmehr, Mussa-Ivaldi, and Bizzi (1993) suggested that if a movement was generated through a gradual shift of the equilibrium position toward the target, then from measurements of the force field about the hand at rest, one should be able to predict the direction and magnitude of forces that should be produced by the muscles during the initiation of the reaching movement (Fig. 1). Because the field at rest is not isotropic, and depends on position of the hand, forces measured during initiation of a movement should not point toward the target and be position dependent. These movement initiation forces were measured and it was found that the pattern of forces from measurements at rest agreed with the measured forces during initiation of movement. In other words, during start of movements the equilibrium point of the field had shifted toward the target.

Won and Hogan (1995) went a step further and suggested that during the entire movement, the static component of the field f_c should be similar to that measured when the hand was at rest, i.e., it should converge to an equilibrium position. In their experiment, the hand was displaced from its intended trajectory via a rigid mechanical constraint. It was shown that as the arm was being displaced, it produced forces directed toward the intended trajectory (Fig. 2). We see the notion of stability about a trajectory (Eq. 3) clearly demonstrated in this data. Controller's output during movement is a force field with an equilibrium point moving roughly along the path connecting the start to the target position.

Dynamics of the muscle-reflex system during movement

Katayama and Kawato (1993) noted that the simulations by Flash (1987) had used a magnitude of stiffness K that was approximately three times that measured when the arm was at rest. While the actual stiffness of the arm was a crucial factor in the simulations, its actual value was unknown

and its estimation had been difficult. Bennett et al. (1992) had found that stiffness during a highly practiced movement was significantly less than that measured when the hand was at rest, while Milner (1993) had found a value that was near the rest levels. It seemed clear that accurate measures of the arm's stiffness during motion were required.

Gomi and Kawato (1996) designed a high performance robotic manipulandum and measured the arm's stiffness during motion. They found that the stiffness of the arm was near those measured at rest but was temporally modulated about this level during motion. They used measures of local stiffness to estimate the point of convergence of the static component of field f_c by assuming that the static muscle forces were linearly related to distance from equilibrium. Gomi and Kawato concluded that motion of the arm could not be due to a simple shift of the equilibrium point along the desired trajectory. This suggested that ultimately, control of motion required explicit compensation for dynamics of the limb.

The crucial question in the work of Gomi and Kawato (1996) was how to estimate the null point of a force field from local measures of stiffness. Most if not all of the experimental data on intact muscle-reflex systems describe only the static behavior, as in Eqs. 4 and 6. Gomi and Kawato showed that if the dynamic behavior of the muscle-reflex system is dominated by its static properties, then it is unlikely that the brain can produce a desired movement via a simple shift of the equilibrium point of the system. But what about the dynamic properties of the muscle-reflex system? How do they contribute to control?

Gribble et al. (1998) approached this question by modifying Eq. 4 to include the effect of delayed sensory feedback on recruitment of motorneurons, and dependence of muscle force on velocity of contraction and temporal summation of activations. The result was a muscle-reflex model that as before was controlled via a threshold muscle length, and had a static behavior that remained similar to Eq. 4, but was now a complex dynamical system. Remarkably, it was found that if the threshold lengths of the muscles acting on a simulated two-joint arm were shifted along a smooth desired trajectory to the target, the resulting motion was also a smooth trajectory. Furthermore, the local stiffness of the system about the actual trajectory was very similar to that reported by Gomi and Kawato (1996). This suggested that the dynamical behavior of the muscle-reflex system was a crucial element in compensating for the arm's dynamic, and that the input to the system might change rather simply from a starting location to a desired endpoint in order to produce a smooth hand trajectory.

It remains to be seen whether the muscle-reflex model proposed by Gribble et al. (1998) has a dynamical behavior that can be valided by experiments on the actual system. If this is the case, then it will be an intriguing example of how the design of the low-level control system and the actuators can simplify computational problems in control of the limb (see GEOMETRICAL PRINCIPLES IN MOTOR CONTROL and FROG WIPING REFLEX).

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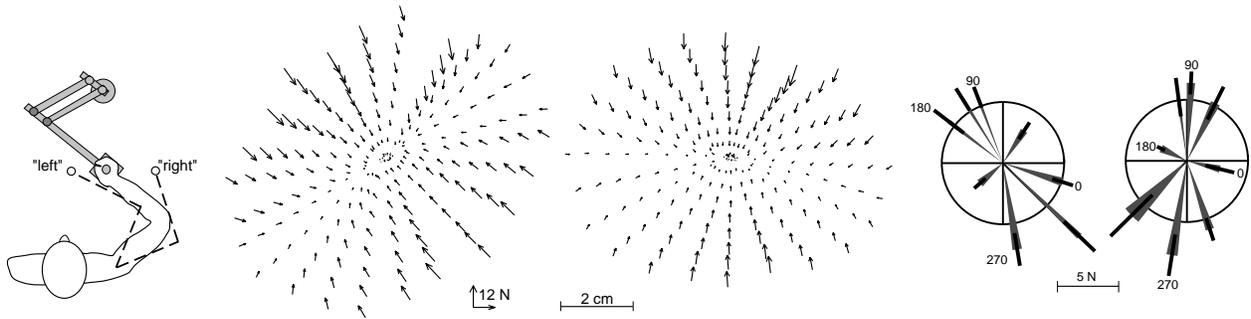
Figure 1. Subject was seated in front of a robotic arm and instructed to hold the handle at either the "right" or "left" configuration. Robot slowly displaced the hand from the origin and measured restoring forces. These forces represent the static component of the force field produced by the muscles, i.e., the postural field. Note the anisotropic shape. Now subjects are told to reach to a target. For randomly selected targets, the robot prevents initiation of the movement (applies a break) and measures the force that the subject is generating in order to make the movement. The magnitude and standard deviations of the movement-related forces are plotted for targets at 0, 45, ..., 315 degrees. The magnitude and direction of movement-related forces are in agreement with the hypothesis that movement is generated through a shift of the equilibrium position of the postural force field toward the target. (From Shadmehr, Mussa-Ivaldi, and Bizz, 1993)

Figure 2. The hand is making a reaching movement while holding the robot. In some cases, the movement is perturbed by forcing the hand to travel in an arc (gray arrows connect the unperturbed trajectory with the perturbed trajectory, similar points in time are connected). Black arrows are the vector difference of the forces between the two trajectories. These forces cause a strong tendency to return the hand to the unconstrained path. (From Won and Hogan, 1995)

7