

## Supplementary materials

### Online processing of uncertain information in visuomotor control

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In the main document, we used the framework of state estimation to consider the problem of reacting to environmental changes. In that framework, the stream of predictions about the near future is integrated with the stream of observations. In order to consider how the state estimation approach compares to other approaches, here we present two alternative models.

#### Alternate Hypothesis 1

One hypothesis, called “decision making model”, predicts that reaction time to a stimulus increases as the uncertainty about that stimulus increases. For example, according to Hick’s law (a standard theory regarding latency of the movement), the reaction time of the movement increases as uncertainty increases (Hick, 1952; Hyman, 1953). In a more recent extension of this model (called LATER), the model explains that reaction time is determined by the log likelihood ratio between two choices (Carpenter and Williams, 1995): when the task is to choose one of two options as soon as possible, the latency is described with the equation  $RT = (\theta - S) / r$ , where

$$S = \log \frac{P(E | H_1)}{P(E | H_2)},$$

$\theta$  is a threshold of neural activity, and  $r$  is the rate of the increase of the

neural activity. This equation indicates that the reaction can be made if a neural activity has reached the threshold. In this model, activity linearly increases as time progress, starting from the bias  $S$ . For example, if the likelihood of the sensory stimulus  $E$  for hypothesis 1 was higher than that for hypothesis 2, the subjects reacted sooner. The LATER model has been applied to a large body of data in decision making task (Bichot and Schall, 1999; Kim and Shadlen, 1999; Reddi and Carpenter, 2000).

To apply the LATER model to our data, suppose that reaction to the jumped target is a decision making process: switch the estimated target position from the first target  $T_1$  to the jumped target  $T_2$ . We show the predictions of this model in Fig. S1A. In order to calculate the response, we used the optimal feedback controller when the estimated target position was forced to change from  $T_1$  to  $T_2$  when the decision making about the switch was made. We see that it predicts very different results than what we saw in our data. For a given 2<sup>nd</sup> target uncertainty, reducing the 1<sup>st</sup>

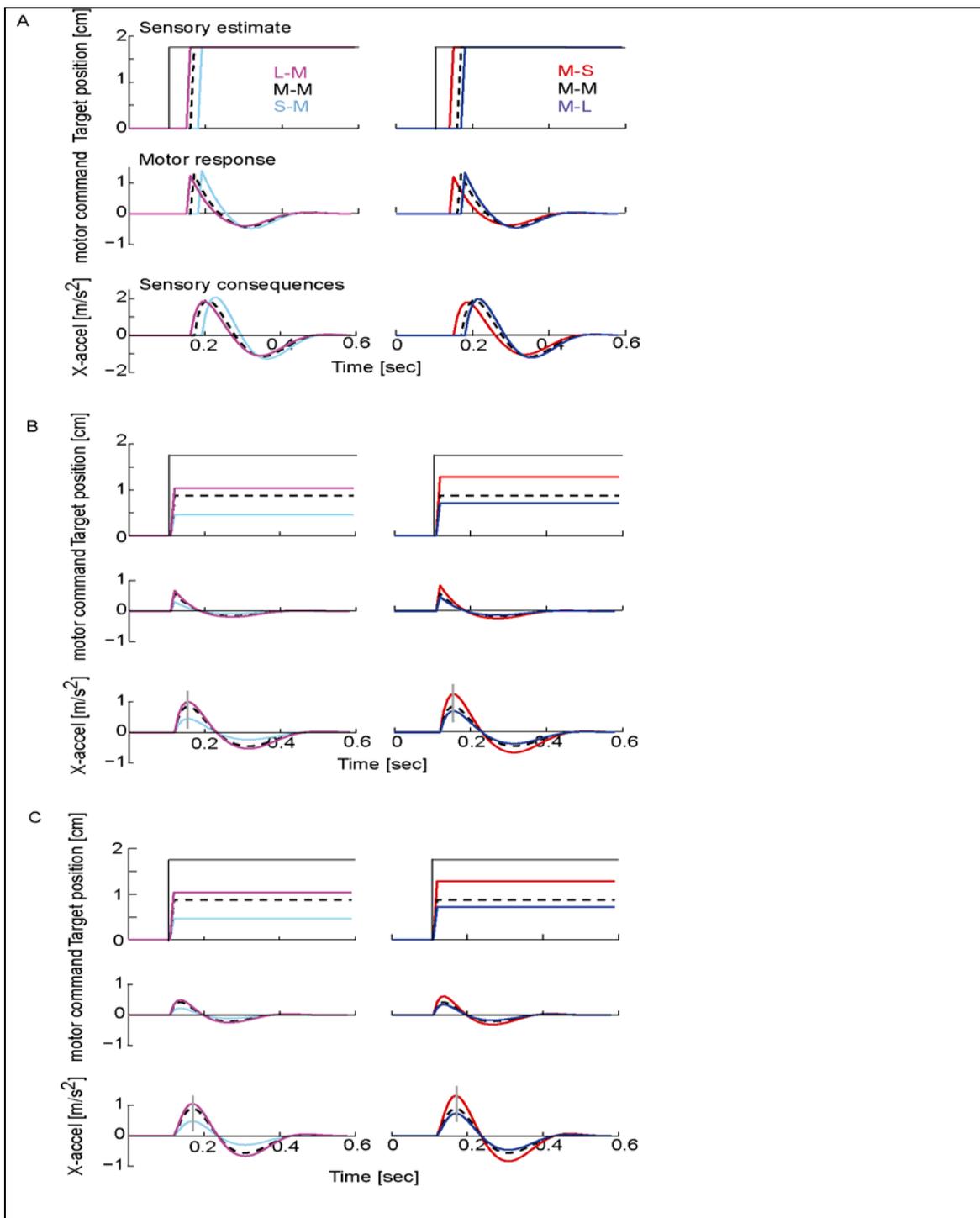
target uncertainty makes the reaction time longer. For a given same 1<sup>st</sup> target uncertainty, reducing the 2<sup>nd</sup> target uncertainty makes the reaction time shorter. However, we did not find any evidence in support of this prediction, as there were no main effects of combinations of target uncertainty in the latency of the reaction to the jumped target ( $F(4,40)=0.81$ ,  $p=0.526$ , mean is 128msec), while the latency of the reaching were clearly altered by the 1<sup>st</sup> target's uncertainty (Figure 1C). Instead of the variation in the latency in the reaction to the jumped target, we saw clear variation in the profile of the response.

### Alternate Hypothesis 2

Let us consider another hypothesis, this time based on Bayesian integration. In this hypothesis, we have a prior observation for the target, and see that it has jumped to a new location. We integrate the new information with the old, and arrive at belief about target position. The weights of the Bayesian integration are determined by the variances of prior and the measurement information. In the Bayesian integration, the integrated target follows this equation:

$$T_{estimated} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} T_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} T_2$$

This prediction was examined with a simulation, as shown in Figure S1B. In order to calculate the response, we used the optimal feedback controller when the estimated target position was forced to change from  $T_1$  to  $T_{estimated}$ . Because the integration is conducted just a single time step, the estimated target does not converge to the jumped target but is simply a weighted average of the 1<sup>st</sup> and 2<sup>nd</sup> targets. We see that the response (X-accel.) has an invariant peak response time, which is indicated with the gray line. As a result, the zero crossing time is also invariant. This characteristic was held even when the motor command was smoothed with a running average ( $u'_{t+1} = 0.5u'_t + 0.5u_t$ ) as shown in Figure S1C. Both are inconsistent with our data because we found that the peak time from the target jump shifted later as the peak response was larger (Figure 4B).



**Figure S1.** Predictions of alternate models. **A.** Decision making model. **B.** Single-step Bayesian integration model. **C.** Smoothed single-step Bayesian integration model. Format is as in Fig. 2C.

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